USN | | | | | | | | | | |

10MCA12

First Semester MCA Degree Examination, January 2011
Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Define power set with examples and show that the power set contains 2^n elements. **(06 Marks)**

b. For any sets A, B and C, prove the following:
i) $A \cap (B - C) = (A \cap B) - C$ ii) $(A - B) \cap (A - C) = A - (B \cup C)$ **(06 Marks)**

c. Let X be the set of all the three digit integers, that is, X = {x is an integer / $100 \le x \le 999$ }. If A_i is the set of numbers in X, whose i^{th} digit is i, compute the cardinality of the set $A_1 \cup A_2 \cup A_3$ **(08 Marks)**

2 a. Define the following: i) Proposition   ii) Tautology   iii) Contradiction
Determine whether the following compound statement is tautology or not:
$[(p \to q) \wedge (p \to q)] \to (p \to r)$ **(08 Marks)**

b. Prove the following by using the laws of logic:
i) $p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$ ii) $[\sim P \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ **(08 Marks)**

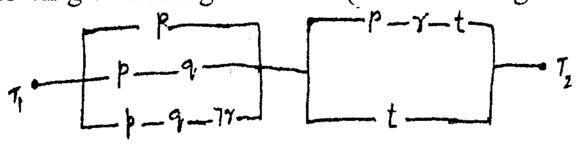
c. Verify the principles of duality for the logical equivalence:
$\sim(p \wedge q) \to \sim p \vee (\sim p \vee q) \Leftrightarrow \sim p \vee q$ **(04 Marks)**

3 a. Using the rules of inference, show that the following argument is valid:

$$\begin{array}{c} p \\ p \to q \\ s \vee r \\ \underline{r \to \neg q} \\ \therefore s \vee t \end{array}$$

(06 Marks)

b. Simplify the following switching networks (without using truth table.) [Refer Fig.Q3(b)]



(06 Marks)

c. Establish the validity of the following argument:

$$\begin{array}{l} \forall x\, [\, p(x) \vee q(x)\,] \\ \exists x\, \neg\, p(x) \\ \forall x\, [\, \neg q(x) \vee r(x)\,] \\ \underline{\forall x\, [\, s(x) \to \neg\, r(x)\,]} \\ \therefore \exists x\, \neg\, s(x) \end{array}$$

(08 Marks)

4 a. Define: i) Well-ordering principle   ii) Principle of mathematical induction. **(06 Marks)**

b. By mathematical induction, prove that $n! \ge 2^{n-1}$, $\forall\, n \ge 1$. **(06 Marks)**

c. A sequence $\{a_n\}$ is defined recursively by $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \ge 2$. Find a_n in explicit form. **(08 Marks)**

Important Note : 1. On completing your answers, c… pulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, … eal to evaluator and /or equations written eg, 42+8 = 50, …'l be treated as malpractice.

5 a. Define Cartesian product of sets, with example.
Show that i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ ii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$ **(08 Marks)**

b. Let $A = \{1, 2, 3, 4\}$. Let R be a relation on A, defined by xRy iff x/y and y=2x. Find
i) R is relation of set of ordered pairs
ii) Draw digraph of R
iii) Determine in degree and out-degree of all vertices. **(06 Marks)**

c. R be an equivalence relation on A and a, b $\in$ A , then show that the following statements are true
i) $a \in [a]$ or $a \in R(a)$
ii) aRb iff $[a]=[b]$ or $R(a) = R(b)$
iii) $[a] \cap [b] \neq \phi$ then $[a] = [b]$ **(06 Marks)**

6 a. Which of the following functions are bijections?
i) $f: R \rightarrow R$ given by $f(x) = 2x - 3$ ii) $g: R \rightarrow R$ given by $g(x) = x^2 - 2$ **(06 Marks)**

b. ABC is an equilateral triangle whose sides are of length 1 cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than ½ cm. **(06 Marks)**

c. Define the invertible function, with an example. Show that a function $f: A \rightarrow B$ is invertible iff it is one to one and onto. **(08 Marks)**

7 a. Let (G, $*$) and (G′, $*'$) be groups with identities e and e′ respectively. If $f: G \rightarrow G'$ is a homomorphism, prove that i) $f(e) = e'$ ii) $f(a^{-1}) = (f(a))^{-1}$, $\forall a \in G$. **(06 Marks)**

b. State and prove the Lagrange's theorem. **(06 Marks)**

c. A binary symmetric channel has probability $p = 0.05$ of incorrect transmission. If the word $C = 011011101$ is transmitted, what is the probability that
i) Single error occurs ii) three errors occur, no two of them consecutive? **(08 Marks)**

8 a. Construct a decoding table (with syndromes) for the group code given by the generator matrix $G=\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$, using the decoding table, decode the following received words:
11110, 11011, 10000, 10101. **(10 Marks)**

b. Show that z_5 is an integral domain. **(05 Marks)**

c. Prove that z_n is a field iff n is a prime. **(05 Marks)**

* * * * *